

# An Investigation in Sequences of Real Numbers Which Converge to $a + b\sqrt{q}$

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The Fibonacci Sequence is one of mathematics most famous sequences; it is found throughout nature and has many applications within pure math. If one were to divide consecutive terms of the Fibonacci sequences, the value approaches  $\frac{1+\sqrt{5}}{2}$ , known as the golden ratio. Using this idea we generalized the concept of the connection between the Fibonacci Sequence and the golden ratio by taking  $a + b\sqrt{q}$ , for  $a, b \in \mathbb{R}$  and  $q \in \mathbb{N}$ , where  $q$  is not a perfect square. We used that value to create a quadratic expression that the a root of the expression is  $a + b\sqrt{q}$ . Then, we would use the quadratic equation to find a sequence that the ratios of terms converge to that value. To accomplish our task we proved that the ratios of the terms in these sequences will converge to the  $a + b\sqrt{q}$  value we started with. We began by considering an explicit case, which we proved using analytical techniques. We generalized this explicit case, and proved it in a similar manner, until we were finally able to prove the most general case; that is, the case for  $a + b\sqrt{q}$ . We continued our exploration by seeing how our sequences related to generating functions, Pascal's triangle, and various geometric shapes.