Elucidating Patterns within a Triangular Array

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There is a popular triangular array in mathematics known as Pascal's triangle. Although Pascal's triangle is named after French mathematician and physicist Blaise Pascal (1623–1662), several other mathematicians knew about the triangle hundreds of years before Pascal. As of today, the triangle appears to have been discovered independently by Chinese mathematician Chia Hsien (1010–1070) and Omar Khayyam (1048–1113). In a recent I.Q. test taken by three teenagers in Canada, a question asked for the next row of numbers, when given the first four rows of Pascal's triangle. The students' answers were different from the numbers in Pascal's triangle. The teacher marked their answers as "wrong," the teenagers complained and they said "without telling us what the triangle is, any five numbers should be acceptable." They had an interesting way of creating the next row, so the teacher challenged them to prove their ideas. These three teenagers have an article published in a well know journal of mathematics by the Mathematical Association of America and their triangle is named the Rascal's triangle. Since the publication of this article, other students from various colleges have been working on this triangle and its properties.

In this presentation, after introducing Rascal's triangle and some of its properties, a new sequence will be defined by adding the diagonal entries in the triangle. The new sequence is called a Fibonacci-like sequence since the Fibonacci sequence can be obtained by adding the diagonal entries from Pascal's triangle. Using the properties of the sequence, a general formula for the Fibonacci-like sequence will be given. This formula uses a summation to add up each diagonal of the triangle to produce our Fibonacci-like sequence. We have observed that the ratio of a term in our Fibonacci-like sequence divided by the previous term appears to converge to one. We will further investigate the proof of convergence by first obtaining a recursive formula for our Fibonacci-like sequence. We also have obtained two polynomials of degree three which, in combination produce the Fibonacci-like sequence. One polynomial produces the odd indices of the sequence and the other polynomial produces the even indices of the sequence.